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HEAT AND MASS TRANSFER IN POROUS AND DISPERSE MEDIA

FILTRATION OF A GAS-CONDENSATE MIXTURE NEAR THE HYDRAULIC FRACTURE

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The problem of filtration of a gas-condensate mixture near the operating well with a hydraulic fracture has been investigated. It was assumed that flow is three-dimensional in the matrix and two-dimensional on the fracture. It has been shown that for steady-state flow, the problem is split into a purely physicochemical problem on phase transitions and a nonlinear partial boundary-value problem for the pressure field. An example of numerical solution for the fracture with a prescribed conductivity field has been given.

Introduction. A formation hydrocarbon mixture for gas-condensate fields with the initial thermobaric conditions is usually in a stable single-phase (gaseous) state. In exploiting the field, the gas is extracted, the formation pressure decreases below the dew point, and a liquid phase, i.e., a condensate, appears. This phenomenon is called retrograde condensation [1, 2]. The fallout of a condensate represents an unfavorable factor from the viewpoint of the efficiency of exploitation of the field: first, the passage of higher hydrocarbons into the liquid phase usually leads to their irreversible loss in the formation; second, the formation of the liquid phase diminishes the productivity of wells. For predicting the consequences of retrograde condensation, it is of interest to analyze the exact solutions of the problem of filtration of a two-phase multicomponent mixture.

Below, we investigate the filtration of a gas-condensate mixture in the vicinity of a hydraulic fracture. Only isothermal flows are considered; therefore, the temperature dependence will be disregarded. Furthermore, capillary effects are assumed negligible.

Formulation of the Problem. We will assume that an M component gas-condensate mixture fills a porous medium in the spatial domain D with a piecewise smooth boundary ∂D. The indices a and b run through values of 1, 2, and 3, corresponding to the ordinal numbers of the spatial coordinates x^a which may not necessarily be Cartesian. The subscripts i, j, and k run through values of 1, ..., and M, corresponding to the numbers of the mixture components. The number of moles of the ith component of the mixture per unit volume is denoted by n_i; the molar weight the ith component is denoted by m_i. A permeable fracture which is described by the two-dimensional smooth surface Γ passes through the domain D. The indices α and β run through values of 1 and 2, corresponding to the ordinal numbers of curvilinear coordinates ξ^α on the surface. In the notation adopted, the surface Γ is described by the equations $x^a = X^a(\xi^\alpha)$. Repeated indices of the coordinates or component numbers indicate sunmation. We will use the notation $g = \det(g_{ab})$, $g_* = \det(g_{\alpha\beta^*})$, $\partial_a = \partial/\partial x^a$, $\partial_{\alpha^*} = \partial/\partial\xi^\alpha$, and $Z_{,i} = \partial Z/\partial n_i$; ∇_a and $\nabla_{\alpha*}$ are the covariant derivatives in space and on the surface Γ (Levi–Civita connectivity for the metrics g_{ab} and $g_{\alpha\beta^*}$ respectively [3]). The metric g_{ab} ($g_{\alpha\beta^*}$) can be used for raising and lowering the indices of tensor fields in the domain D (on the surface Γ) [4].

Let $n_{i\mathrm{g}} = n_{i\mathrm{g}}(t, x^a)$, $n_{i\mathrm{c}} = n_{i\mathrm{c}}(t, x^a)$, and $n_{i\mathrm{g}*} = n_{i\mathrm{g}*}(t, \xi^\alpha)$, $n_{i\mathrm{c}*} = n_{i\mathrm{c}*}(t, \xi^\alpha)$. Accordingly let $s_\mathrm{g} = s_\mathrm{g}(t, x^a)$, $s_\mathrm{c} = s_\mathrm{c}(t, x^a)$, $(s_\mathrm{g} + s_\mathrm{c} = 1)$, and $s_{\mathrm{g}*} = s_{\mathrm{g}*}(t, \xi^\alpha)$, $s_{\mathrm{c}*} = s_{\mathrm{c}*}(t, \xi^\alpha)$, $(s_{\mathrm{g}*} + s_{\mathrm{c}*} = 1)$.

For the mixture in question we have determined the free energy per unit volume $f = f(n_i)$ dependent on the molar densities of the components n_i. In applications, we compute the function $f = f(n_i)$ on the basis of semiempirical

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equations of state [1, 2]. The chemical potential of the mixture component $\kappa_i = f_{,i}$ and the hydrostatic pressure $p = n_i\kappa_i - f$ can be calculated using the free energy. We recall the Gibbs–Duhem relation

$$dp = n_i d\kappa_i \,, \tag{1}$$

used below in a number of conclusions.

From the densities of the components, we compute their chemical potentials in the gas and the condensate $\kappa_{ig} = \kappa_i(n_{jg})$ and $\kappa_{ic} = \kappa_i(n_{jc})$ and the pressures in the phases $p_g = p(n_{ig})$ and $p_c = p(n_{ic})$ in the matrix. We compute the chemical potentials of the components in the gas and the condensate $\kappa_{ig*} = \kappa_i(n_{jg*})$ and $\kappa_{ic*} = \kappa_i(n_{jc*})$ and the pressures in the phases $p_{g*} = p(n_{ig*})$ and $p_{c*} = p(n_{ic*})$ in the fracture. We will assume that the conditions of local thermodynamic equilibrium between the phases are observed in the matrix and the fracture

$$\kappa_{ig} = \kappa_{ic} \,, \quad p_g = p_c \,, \tag{2}$$

$$\kappa_{ig*} = \kappa_{ic*} \,, \quad p_{g*} = p_{c*} \tag{3}$$

in the absence of capillary forces.

We assume that the porous medium is homogeneous and isotropic and the porosity factor m is pressure-independent. Let the opening of the fracture be prescribed by a smooth field on the surface Γ: $h_* = h_*(\xi^\alpha)$, and the medium filling the fracture be characterized by its own constant porosity factor m_*. Then we can write the expression for the total free energy of the mixture

$$F = m \int\limits_D \Big(s_g f(n_{ig}) + s_c f(n_{ic}) + (s_g n_{ig} + s_c n_{ic})\, m_i\varphi \Big) g\, dx^1 dx^2 dx^3 \tag{4}$$

$$+ m_* \int\limits_\Gamma \Big(s_{g*} f(n_{ig*}) + s_{c*} f(n_{ic*}) + (s_{g*} n_{ig*} + s_{c*} n_{ic*})\, m_i\varphi \Big) h_* g_* d\xi^1 d\xi^2 \,,$$

where $\varphi = \varphi(x^a)$. In filtration of the gas-condensate mixture, the conditions of local conservation of the components in the matrix

$$m\partial_t (s_g n_{ig} + s_c n_{ic}) + \nabla_a I_i^a = 0 \tag{5}$$

and in the fracture

$$m_* h_* \partial_t (s_{g*} n_{ig*} + s_{c*} n_{ic*}) + \nabla_{a*} I_{i*}^a + \big[I_i^a l_a \big] = 0 \tag{6}$$

must be observed. In Eqs. (5) and (6), $\big[I_i^a l_a \big]$ denotes the jump in the quantity computed as the difference between the value from the side to which the vector l_a is directed and the value from the opposite side.

We assume that the boundary $\partial\Gamma$ on the surface Γ represents a piecewise smooth curve. Let k^α be a unit inner normal to $\partial\Gamma$ in the geometry of the surface Γ, ∂s be a measure at $\partial\Gamma$, $\gamma_1 = \partial\Gamma \cap \partial D$, and $\gamma_2 = \partial\Gamma - \gamma_1$. Also, we assume that the internal flows I_{i*}^α vanish on the γ_2 curve.

Using the dynamic equations (5) and (6), phase-equilibrium conditions (2) and (3), and the assumptions made, we can compute the time derivative of the total free energy of the mixture (4):

$$\frac{dF}{dt} = \Sigma_1 + \Sigma_2 \,, \tag{7}$$

$$\Sigma_1 = \int\limits_{\partial D} K_a I_i^a (\kappa_i + m_i\varphi)\, dA + \int\limits_{\gamma_1} k_\alpha I_{i*}^\alpha (\kappa_{i*} + m_i\varphi)\, ds \,,$$

$$\Sigma_2 = \int\limits_D I_i^a \partial_a (\kappa_i + m_i \varphi)\, g dx^1 dx^2 dx^3 + \int\limits_\Gamma \Big(I_{i*}^\alpha \partial_{\alpha *} (\kappa_i + m_i \varphi) + (\kappa_i - \kappa_{i*})\, [I_i^a l_a] \Big) g_* d\xi^1 d\xi^2 . \tag{8}$$

The term Σ_1 in expression (7) describes the change in the free energy due to the flow through the boundary of the domain, and Σ_2 describes the same but within the domain. For isothermal processes, an analog of the well-known condition of nonnegativeness of the entropy production is the inequality

$$\Sigma_2 \le 0 . \tag{9}$$

For the flows, we take the ordinary assumption of the transfer of the components due to the phase flow in the pores

$$I_i^a = n_{i\mathrm{g}} u_\mathrm{g}^a + n_{i\mathrm{c}} u_\mathrm{c}^a , \tag{10}$$

$$I_{i*}^\alpha = h_* \Big(n_{i\mathrm{g}*} u_{\mathrm{g}*}^\alpha + n_{i\mathrm{c}*} u_{\mathrm{c}*}^\alpha \Big) . \tag{11}$$

Furthermore, we take the conditions of equality of the chemical potentials in the fracture and the matrix

$$\kappa_{i*} = \kappa_i \big|_\Gamma . \tag{12}$$

Relations (12), in particular, yield that the densities of the components in the phases and the pressures coincide for the mixture in the matrix and the fracture

$$n_{i\mathrm{g}*} = n_{i\mathrm{g}} \big|_\Gamma , \quad n_{i\mathrm{c}*} = n_{i\mathrm{c}} \big|_\Gamma , \quad p_* = p \big|_\Gamma . \tag{13}$$

At the same time, the saturations in the fracture and the matrix can generally differ: $s_{\mathrm{g}*} \ne s_\mathrm{g} \big|_\Gamma$.

Using relations (1), (10), (11), and (13), we reduce expression (8) to the form

$$\Sigma_2 = \int\limits_D \Big(u_\mathrm{g}^a (\partial_a p + \rho_\mathrm{g} \partial_a \varphi) + u_\mathrm{c}^a (\partial_a p + \rho_\mathrm{c} \partial_a \varphi) \Big) g dx^1 dx^2 dx^3 \tag{14}$$

$$+ \int\limits_\Gamma \Big(u_{\mathrm{g}*}^\alpha (\partial_{\alpha *} p + \rho_\mathrm{g} \partial_{\alpha *} \varphi) + u_{\mathrm{c}*}^\alpha (\partial_{\alpha *} p + \rho_\mathrm{c} \partial_{\alpha *} \varphi) \Big) g_* d\xi^1 d\xi^2 ,$$

where $\rho_\mathrm{g} = m_i n_{i\mathrm{g}}$ and $\rho_\mathrm{c} = m_i n_{i\mathrm{c}}$. From (14), it is seen that the validity of Darcy's law for the phases in the matrix and the fracture is sufficient for the inequality (9) to hold:

$$u_\mathrm{g}^a = -\, k f_\mathrm{g} \mu_\mathrm{g}^{-1} g^{ab} (\partial_b p + \rho_\mathrm{g} \partial_b \varphi) , \tag{15}$$

$$u_\mathrm{c}^a = -\, k f_\mathrm{c} \mu_\mathrm{c}^{-1} g^{ab} (\partial_b p + \rho_\mathrm{c} \partial_b \varphi) , \tag{16}$$

$$u_{\mathrm{g}*}^\alpha = -\, k_* f_{\mathrm{g}*} \mu_\mathrm{g}^{-1} g^{ab} (\partial_{\beta *} p + \rho_\mathrm{g} \partial_{\beta *} \varphi) , \tag{17}$$

$$u_{\mathrm{c}*}^\alpha = -\, k_* f_{\mathrm{c}*} \mu_\mathrm{c}^{-1} g^{ab} (\partial_{\beta *} p + \rho_\mathrm{c} \partial_{\beta *} \varphi) . \tag{18}$$

It is assumed that the relative phase permeabilities are prescribed as functions of saturation of the condensate. The absolute permeability of the matrix k is coordinate-independent in accordance with the assumption of homogeneity of the porous medium. At the same time, the absolute permeability of the medium filling the fracture generally represents the function on the surface Γ: $k_* = k_*(\xi^\alpha)$.

Relations (2), (10), (11), (13), and (15)–(18) close the dynamic problem (5) and (6). There can be different physically substantive formulations of the problem that differ in the geometry of the domain D and the surface Γ and in boundary and initial conditions.

It is noteworthy that expressions (12) and (15)–(18) are not the only possible set of relations closing the problem and compatible with condition (9). Thus, e.g., there are complicated filtration models where the expressions for filtration rates are dependent on the pressure gradient nonlinearly. The model proposed is the most simple from the viewpoint of the analytical form of the governing relations. At the same time, it is consistent with numerous laboratory and field observations.

We investigate the stationary solutions of the problem which correspond to steady-state filtration flows of a gas-condensate mixture. It is well known that the problem of steady-state filtration of the gas-condensate mixture is integrated in quadratures in one-dimensional and plane cases [5–8]. The properties of exact solutions can be used for interpretation of the stationary investigations of gas-condensate mixtures and prediction of the productivity of wells [9]. Below, it is shown that the well-known method of solution of one-dimensional and two-dimensional cases [5–8] is generalized to the three-dimensional problem of filtration in the vicinity of a hydraulic fracture.

In the stationary case, the equations of conservation of the components (5) and (6) are reduced to the conditions for the component fluxes

$$\nabla_a I_i^a = 0 , \tag{19}$$

$$\nabla_{\alpha *} I_{i*}^\alpha + \left[I_i^a l_a \right] = 0 . \tag{20}$$

We will assume that the surface Γ lies within the domain D. We assume that the two-dimensional boundary ∂D of the domain D falls into two piecewise smooth surfaces S_1 and S_2. A constant pressure p_r corresponding to the formation pressure is prescribed on the surface S_1, and the condition of the flow through this surface is specified on the surface S_2. Since the surface Γ describes the hydraulic fracture, it must be geometrically connected with the operating well. In the formulation in question, the trajectory of the well-bore is described by a certain curve L on the surface Γ. We assume that a constant pressure p_w corresponding to the bottom-hole pressure is prescribed on the curve L. Gravitational forces will be disregarded.

To analyze Eq. (19) it is convenient to select a coordinate system in space such that $x^1 = p$ and $g_{1\alpha} = 0$ and the coordinates x^2, x^3 on the fracture are coincident with the intrinsic coordinates of the surface Γ: $x^2 = \xi^1$ and $x^3 = \xi^2$. We can always determine such a system: it is sufficient to set the coordinates x^2, x^3 constant along the streamlines (i.e., along the lines of the pressure-gradient field). However, it should be borne in mind in computations that mapping of the domain D onto the corresponding domain in coordinates p, ξ^α is two-sheeted, since the streamline can approach the fracture on both sides for the same values of the parameters ξ^1 and ξ^2.

In the coordinate system selected, the fracture is described by the equation $p = p_*(\xi^\alpha)$, the boundary of the domain ∂D is described by the equation $p = p_\mathrm{r}$, and the well L is described by the equation $p = p_\mathrm{w}$; the metric form in space, by definition, appears as

$$ds^2 = P^2 dp^2 + \sigma_{\alpha\beta} d\xi^\alpha d\xi^\beta . \tag{21}$$

The metric form on the fracture Γ is calculated from the formula $g_{\alpha\beta *} = P^2 \partial_{\alpha *} p_* \partial_{\beta *} p_* + \sigma_{\alpha\beta}$.

Using the metric (21), we can rewrite the system of equations (19) in the selected coordinate system

$$\frac{\partial}{\partial p}\left(\frac{\sigma^{1/2}}{P} k \left(B_\mathrm{g} c_{i\mathrm{g}} + B_\mathrm{c} c_{i\mathrm{c}} \right) \right) = 0 , \tag{22}$$

$$B_g = f_g \mu_g^{-1} n_g\,, \quad B_c = f_c \mu_c^{-1} n_c\,; \quad n_g = \sum_{i=1}^{M} n_{ig}\,, \quad n_c = \sum_{i=1}^{M} n_{ic}\,; \quad c_{ig} = n_{ig}/n_g\,,$$

$$c_{ic} = n_{ic}/n_c\,; \quad \sigma = \det(\sigma_{\alpha\beta})\,.$$

The system of equations (22) has M first integrals

$$\frac{\sigma^{1/2}}{P} k\,(B_g c_{ig} + B_c c_{ic}) = q_i\,(\xi^\alpha)\,. \tag{23}$$

The right-hand sides in these equations represent the component flows arriving at the considered region D through the boundary of the domain ∂D. In the case where the formation pressure p_r is higher than or equal to the saturation pressure p_D, the condensate at the boundary ∂D is absent ($B_c = 0$), and the right-hand sides are clearly in proportion to the composition of the gas, i.e., to the composition of the initial formation mixture c_{i0}:

$$q_i = c_{i0} q\,(\xi^\alpha)\,. \tag{24}$$

If the formation pressure is lower than the saturation pressure p_D and, consequently, there is a condensate at the boundary ∂D, we will assume, as previously, that relations (24) hold, however the set of concentrations c_{i0} is interpreted as the composition of the moving part of the formation mixture.

The total flux to the well is computed as the integral along the boundary ∂D of the flux density

$$Q = \int_{\partial D} P^{-1} k\,(B_g + B_c)\,\sigma^{1/2} d\xi^1 d\xi^2 = \int_{\partial D} q\,(\xi^\alpha)\,d\xi^1 d\xi^2\,. \tag{25}$$

Using (24) we reduce the set of integrals (23) to the form

$$A_g c_{ig} + A_c c_{ic} = c_{i0}\,, \tag{26}$$

$$A_g = \sigma^{1/2} P^{-1} q^{-1} k B_g\,; \quad A_c = \sigma^{1/2} P^{-1} q^{-1} k B_c\,.$$

Expressions (26) have the form of balance relations in disintegration of the mixture of composition c_{i0} into the gas and the condensate with compositions c_{ig} and c_{ic} respectively. Thus, the representation

$$A_g = 1 - W\,, \quad A_c = W \tag{27}$$

is true; in it, the function $W = W(p)$ representing the mole fraction of the condensate in the mixture of average composition c_{i0} can be determined, irrespective of the filtration problem, either from experiment or by calculation using one semiempirical equation of state [1, 2]. Analogously we can find all the characteristics of the gas and the condensate, c_{ig}, c_{ic}, n_g, n_c, μ_g, and μ_c, as functions of the pressure p.

From relations (27), we derive an equality containing no metric coefficients:

$$f_c/f_g = W \mu_c n_g\,(1 - W)^{-1}\,\mu_g^{-1} n_c^{-1}\,. \tag{28}$$

There is the known function of the pressure p on the right-hand side of equality (28) and the known function of the saturation of the condensate in the matrix s_c. Thus, we can interpret relation (28) as the equation determining the saturation s_c as a function of the pressure p.

The above reasoning shows that we can obtain the dependences $B_\mathrm{g} = B_\mathrm{g}(p)$ and $B_\mathrm{c} = B_\mathrm{c}(p)$ even before solving the filtration problem proper, which must give the spatial distribution of the parameters of the mixture. Introducing the notation $\Phi = \Phi(p) + B_\mathrm{c}(p)$, we easily derive, from (19), the elliptic equation for the matrix pressure

$$0 = k^{-1} \sum_{i=1}^{M} \nabla_a I_i^a = g^{ab} \nabla_a \left(\Phi(p) \nabla_b p\right) . \tag{29}$$

Now we investigate the system of equations (20). Using the results obtained on the component fluxes in the matrix, we can transform this system as follows:

$$g_*^{\alpha\beta} k^{-1} \nabla_{\alpha*} \left(k_* h_* \left(B_{\mathrm{g}*} c_{i\mathrm{g}} + B_{\mathrm{c}*} c_{i\mathrm{c}}\right) \nabla_{\beta*} p\right) = - \Phi c_{i0} \left[l^a \partial_a p\right] , \tag{30}$$

$$B_{\mathrm{g}*} = f_{\mathrm{g}*} \mu_\mathrm{g}^{-1} n_\mathrm{g} ; \quad B_{\mathrm{c}*} = f_{\mathrm{c}*} \mu_\mathrm{c}^{-1} n_\mathrm{c} .$$

The right-hand side of the system of equations (30) is in proportion to a constant c_{i0} vector; therefore, we can seek such a solution of the problem in which the expression under the derivative on the right-hand side of the system, dependent on the nomber. of chemical component, is also in proportion to the c_{i0} vector. Analogously to (26), we obtain the balance relations for the concentrations

$$A_{\mathrm{g}*} c_{i\mathrm{g}} + A_{\mathrm{c}*} c_{i\mathrm{c}} = c_{i0} , \quad A_{\mathrm{g}*} = \Phi_*^{-1} B_{\mathrm{g}*} , \quad A_{\mathrm{c}*} = \Phi_*^{-1} B_{\mathrm{c}*} , \quad \Phi_* = B_{\mathrm{g}*} + B_{\mathrm{c}*} .$$

Next, analogously to (27), the relations $A_{\mathrm{g}*} = 1 - W$ and $A_{\mathrm{c}*} = W$ are true. Consequently, there is an analog of Eq. (28) for determination of the saturation of the condensate in the fracture $s_{\mathrm{c}*}$ as a function of the pressure p:

$$f_{\mathrm{c}*} / f_{\mathrm{g}*} = W \mu_\mathrm{c} n_\mathrm{g} (1 - W)^{-1} \mu_\mathrm{g}^{-1} n_\mathrm{c}^{-1} .$$

Thus, the quantity Φ_* has been determined as a function of the pressure p. Summation of the equations of system (30) over component numbers yields a differential relation for the pressure p, which has been determined on the surface Γ. It is the internal boundary condition for problem (29):

$$g_*^{\alpha\beta} k^{-1} \nabla_{\alpha*} \left(k_* h_* \Phi_* \nabla_{\beta*} p\right) + \Phi \left[l^a \partial_a p\right] = 0 . \tag{31}$$

In accordance with what has been stated above, the boundary conditions

$$p \,|_{S_1} = p_\mathrm{r} , \quad \lambda^a \partial_a p \,|_{S_2} = 0 , \tag{32}$$

$$p \,|_{L} = p_\mathrm{w} \tag{33}$$

are imposed on the pressure p in addition to condition (31).

We note the important particular case where the fracture has very high conductivity ($(k_*/k) \to +\infty$). The problem is reduced to Eq. (29), boundary conditions (32), and the following supplementary boundary condition replacing conditions (31) and (33):

$$p \,|_{\Gamma} = p_\mathrm{w} . \tag{34}$$

We give an example of numerical solution of the filtration problem for steady-state flow of a gas-condensate mixture. We will use the composition of the gas-condensate mixture, which corresponds to the second object of the Karachaganak oil and gas-condensate field (Republic of Kazakhstan). The composition of the mixture in mole fractions

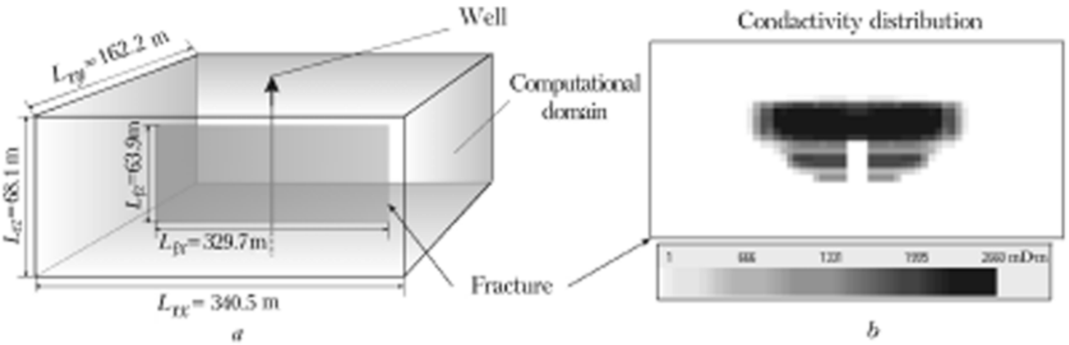

Fig. 1. Geometry of the computational domain (a) and the conductivity of the fracture (b).

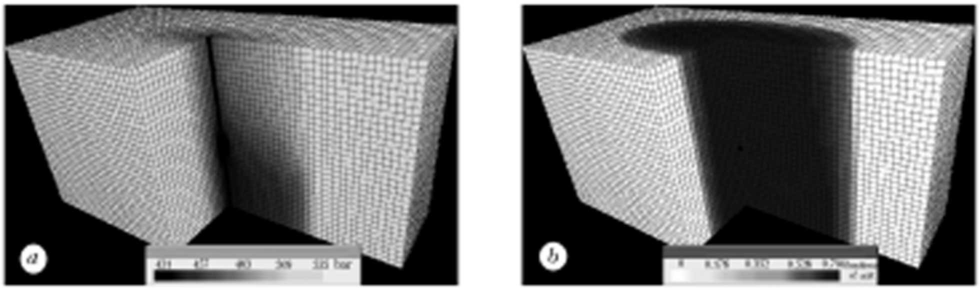

Fig. 2. Fields of pressure (a) and saturation of the condensate (b) in the matrix.

is as follows: $c_{N_2} = 0.0103$, $c_{CO_2} = 0.0462$, $c_{H_2S} = 0.0432$, $c_{CH_4} = 0.6269$, $c_{C_2H_6} = 0.0822$, $c_{C_3H_8} = 0.0308$, $c_{nC_4H_{10}} = 0.0062$, $c_{iC_4H_{10}} = 0.0103$, $c_{C_5} = 0.0285$, $c_{C_6} = 0.0149$, and $c_{C_{7+}} = 0.1005$.

We take the formation pressure $p_r = 535$ bar and the bottom-hole pressure $p_w = 415$ bar. The formation pressure is close to a saturation pressure of 530 bar (1 bar = 10^5 Pa). The thermodynamic characteristics and phase transitions are calculated from the Peng–Robinson equation of state [1, 2]. The gas and condensate viscosities are taken to be constant: $\mu_g = 2.3 \cdot 10^{-5}$ Pa·sec and $\mu_c = 4.9 \cdot 10^{-4}$ Pa·sec.

Numerical modeling is carried out for a rectangular plane fracture with a prescribed conductivity field ($k_* h_*$) for fixed dimensions of the computational domain (see Fig. 1). The upper and lower sides of the computational domain are considered impermeable; a value of pressure equal to that of formation pressure is taken on the lateral sides. The permeability of the matrix is set equal to 1 mD.

The parameters of the finite-difference grid used are as follows:

(1) the three-dimensional computational domain with dimensions 340.5 × 162.2 × 68.1 m is subdivided into 63 × 30 × 32 cells;

(2) the two-dimensional fracture with dimensions 329.7 × 63.9 is subdivided into 61 × 30 cells.

In calculations, we take the following dependences of the relative phase permeabilities on pressure:

$$f_g = (s_g - s_{g1})^a / (1 - s_{g1})^a \,, \quad f_c = (s_c - s_{c1})^b / (1 - s_{c1})^b \,.$$

We use different relative phase permeabilities: $a = 2$, $b = 3$, $s_{g1} = 0.08$, and $s_{c1} = 0.12$ in the matrix and $a = 2$, $b = 2$, $s_{g1} = 0$, and $s_{c1} = 0$ on the fracture. From the calculation results, the gas flow rate is 661,000 m^3/day.

Figures 2 and 3 give the distribution of the pressure and saturation of the condensate. An important feature of the solution is the formation of a condensate bank near the fracture with a considerable pressure gradient in this region.

Conclusions. It has been shown that the problem of filtration of a gas-condensate mixture in the vicinity of the hydraulic fracture is reduced to a nonlinear elliptic equation with boundary conditions involving the nonlinear elliptic operator on the fracture (see (31)). Such an equation can generally be solved only by numerical methods. None-

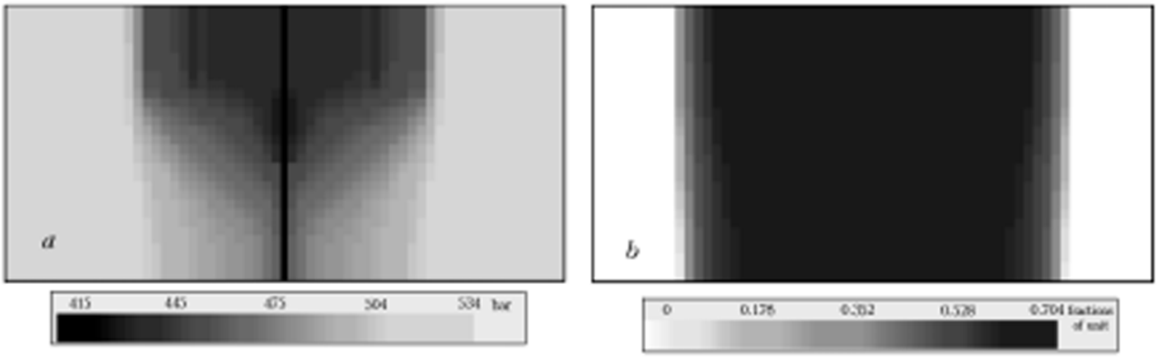

Fig. 3. Distribution of the: a) pressure and b) saturation of the condensate fracture.

theless, the problem obtained for pressure is much more simple than the initial problem (19) and (20) containing the unknown compositions of the phases and saturations. We emphasize that Eq. (29) and (31) have been written in an invariant form independent of the coordinate systems used in space and on the surface Γ, although we considered a wholly concrete coordinate system (see (21)) in deriving these equations.

The constructed model of filtration of a gas-condensate mixture in the vicinity of a hydraulic fracture makes it possible to predict both the productivity of the well and the distribution of the mixture parameters in the matrix and the fracture. Such prediction can be useful in simulating hydraulic fracturing.

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NOTATION

A_g, A_c, A_{g*}, and A_{c*}, auxiliary variables; B_g, B_c, B_{g*}, and B_{c*}, auxiliary variables, mole·sec/(m^2·kg); c_{i0}, concentration of the ith component in the formation mixture; c_{ig} and c_{ic}, concentrations of the ith component in the gas and the condensate; D, spatial domain; ∂D, boundary of the domain D; $\partial\Gamma$, boundary of the surface Γ; ds, measure at $\partial\Gamma$, m; dA, surface element, m^2; f, free energy of the mixture per unit volume, Pa; F, total free energy of the mixture, J; f_g and f_c, coefficients of relative phase permeabilities of the gas and the condensate in the matrix; f_{g*} and f_{c*}, coefficients of relative phase permeabilities of the gas and the condensate in the fracture; g, metric-tensor determinant in space; g_*, metric-tensor determinant in the surface; g_{ab}, covariant components of the metric tensor in space; $g_{\alpha\beta*}$, covariant components of the metric tensor on the surface Γ; h_*, fracture opening, m; I_i^a, flux of the ith component in the matrix, mole/(sec·m^2); I_{i*}^{α}, flux of the ith component in the fracture, mole/(sec·m); k and k_*, coefficients of absolute permeability of the matrix and the fracture, m^2; K_a, unit inner normal to ∂D; k^{α}, unit inner normal to $\partial\Gamma$; l_a, inner normal to the surface Γ; L, trajectory of the well-bore; M, number of components in the mixture; m, porosity factor in the matrix; m_*, porosity factor in the fracture; m_i, molar weight of the ith component of the mixture, kg/mole; n_g and n_c, molar densities of the gas and the condensate, mole/m^3; n_i, molar density of the ith component of the mixture, mole/m^3; m_{ig} and n_{ic}, densities of the components of the gas and the condensate in the matrix, mole/m^3; n_{ig*} and n_{ic*}, densities of the components of the gas and the condensate in the fracture, mole/m^3; P, metric pressure coefficient, m/Pa; p and p_*, hydrostatic pressures in the matrix and the fracture, Pa; p_D, saturation pressure, Pa; p_g and p_c, pressures of the gas and the condensate in the matrix, Pa; p_{g*} and p_{c*}, pressures of the gas and the condensate in the fracture, Pa; p_r, formation pressure, Pa; p_w, bottom-hole pressure, Pa; Q, total flux to the well, mole/sec; q, flow rate of the mixture, mole/(m^2·sec); q_i, flow rate of the ith component of the mixture, mole/(m^2·sec); s_g and s_c, saturations of the gas and the condensate in the matrix; s_{g*} and s_{c*}, saturations of the gas and the condensate in the fracture; s_{g1} and s_{c1}, mobility thresholds for the gas and the condensate; S_1 and S_2, auxiliary surfaces; t, time, sec; u_g^a and u_c^a, rates of filtration of the gas and the condensate in the matrix, m/sec; u_{g*}^{α} and u_{c*}^{α}, rates of filtration of the gas and the condensate in the fracture, m/sec; W, mole fraction of the condensate in the mixture of average composition c_{i0}; x^a, spatial coordinates; γ_1, intersection of the fracture surface and the boundary; γ_2, part of the fracture boundary located within the domain D; κ_i and κ_{i*}, chemical potentials of the ith component of the mixture in the matrix and the fracture, kg·m^2/(sec^2·mole); κ_{ig} and κ_{ic}, chemical potentials in the matrix of the ith component of the gas and the condensate, kg·m^2/(sec^2·mole); κ_{ig*} and κ_{ic*}, chemical potentials in the fracture in the *i*th component of the gas and the condensate, kg·m^2/(sec^2·mole); λ^a, normal to the surface S_2; μ_g and μ_c, shear viscosities of the gas and the condensate, Pa·sec; ξ^α, curvilinear coordinates on the two-dimensional surface; ρ_g and ρ_c, mass densities of the gas and the condensate, kg/m^3; σ, determinant of $\sigma_{\alpha\beta}$; $\sigma_{\alpha\beta}$, part of the metric tensor; Σ_1 and Σ_2, auxiliary variables, J/sec; φ, gravitational potential, m^2/sec^2; Φ and Φ_*, auxiliary functions. Subscripts and superscripts: *a* and *b*, for spatial coordinates; *i*, *j*, and *k*, for the components of the mixture; α and β, for curvilinear coordinates; *x*, *y*, *z*, coordinate axes; g, gas; c, condensate; f, fracture; r, computational domain (region); w, well.

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